

CBSE Class 11 Mathematics

Important Questions

Chapter 8

Binomial Theorem

1 Marks Questions

1. What is The middle term in the expansion of $(1+x)^{2n+1}$

Ans. Since $(2n+1)$ is odd there is two middle term

$$\text{i.e. } {}^{2n+1}C_n x^{n+1} \text{ and } {}^{2n+1}C_{n+1} x^n$$

2. When n is a positive integer, the no. of terms in the expansion of $(x+a)^n$ is

Ans. The no. of terms in the expansion of $(x+a)^n$ is one more than the index n . i.e. $(n+1)$.

3. Write the general term $(x^2 - y)^6$

$$\text{Ans. } T^{r+1} = {}^6C_r (x^2)^{6-r} \cdot (-y)^r$$

$$= {}^6C_r (x)^{12-2r} \cdot (-1)^r \cdot (y)^r$$

4. In the expansion of $\left(x + \frac{1}{x}\right)^6$, find the 3rd term from the end

Ans. 3rd term from end = $(6-3+2)^{\text{th}}$ term from beginning

$$\text{i.e } T_5 = {}^6C_4(x)^{6-4} \cdot \left(\frac{1}{x}\right)^4$$

$$= {}^6C_4 x^2 \cdot x^{-4}$$

$$= 15x^{-2}$$

$$= \frac{15}{x^2}$$

5. Expand $(1+x)^n$

$$\text{Ans. } (1+x)^n = 1 + {}^nC_1(x)^1 + {}^nC_2(x)^2 + {}^nC_3(x)^3 + \dots x^n$$

6. The middle term in the expansion of $(1+x)^{2n}$ is

$$\text{Ans. } {}^{2n}C_n \cdot x^n$$

7. Find the no. of terms in the expansions of $(1-2x+x^2)^7$

$$\text{Ans. } (1-2x+x^2)^7$$

$$= (x^2 - 2x + 1)^7$$

$$= [(x-1)^2]^7$$

$$= (x-1)^{14}$$

No. of term is 15

8. Find the coeff of x^5 in $(x+3)^9$

$$\text{Ans. } T_{r+1} = {}^9C_r (x)^{9-r} \cdot (3)^r$$

$$\text{Put } 9-r=5$$

$$r=4$$

$$T_5 = {}^9C_4 (x)^5 \cdot (3)^4$$

$$\text{Coeff of } x^5 \text{ is } {}^9C_4 (3)^4$$

9. Find the term independent of $x \left(x + \frac{1}{x}\right)^{10}$

$$\text{Ans. } T_{r+1} = {}^{10}C_r (x)^{10-r} \cdot \left(\frac{1}{x}\right)^r$$

$$= {}^{10}C_r (x)^{10-r} \cdot (x)^{-r}$$

$$= {}^{10}C_r (x)^{10-2r}$$

$$\text{Put } 10-2r=0$$

$$r=5$$

$$\text{Independent term is } {}^{10}C_5$$

10. Expand $(a+b)^n$

$$\text{Ans. } (a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_n b^n$$



CBSE Class 12 Mathematics

Important Questions

Chapter 8

Binomial Theorem

4 Marks Questions

1. Which is larger $(1.01)^{10,00,000}$ or 10,000

Ans. $(1.01)^{10,00,000} = (1+0.01)^{10,00,000}$

$$= {}^{10,00,000}C_0 + {}^{10,00,000}C_1(0.01) + \text{other positive term}$$

$$= 1 + 10,00,000 \times 0.01 + \text{other positive term}$$

$$= 1 + 10,000$$

$$= 10,001$$

Hence $(1.01)^{10,00,000} > 10,000$

2. Prove that $\sum_{r=0}^n 3^r {}^nC_r = 4^n$

Ans.

$$\sum_{r=0}^n 3^r {}^nC_r = \sum_{r=0}^n {}^nC_r 3^r$$

$$= {}^nC_0 + {}^nC_1 \cdot 3 + {}^nC_2 \cdot 3^2 + \dots + {}^nC_n 3^n$$

$$\left[\because (1+a)^n = 1 + {}^nC_1 a + {}^nC_2 a^2 + {}^nC_3 a^3 + \dots + a^n \right]$$

$$= (1+3)^n$$

$$= (4)^n$$

H.P

3. Using binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25.

Ans. Let $6^n = (1+5)^n$

$$= 1 + {}^n C_1 5^1 + {}^n C_2 5^2 + {}^n C_3 5^3 + \dots + 5^n$$

$$= 1 + 5n + 5^2 \left({}^n C_2 + {}^n C_3 \cdot 5 + \dots + 5^{n-2} \right)$$

$$6^n - 5n = 1 + 25 \left({}^n C_2 + {}^n C_3 \cdot 5 + \dots + 5^{n-2} \right)$$

$$= 1 + 25k \left[\text{where } k = {}^n C_2 + {}^n C_3 \cdot 5 + \dots + 5^{n-2} \right]$$

$$= 25k + 1$$

H.P

4. Find the 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}} \right)^{18}$, $x \neq 0$

Ans. The general term in the expansion of

$$\left(9x - \frac{1}{3\sqrt{x}} \right)^{18} \text{ is}$$

$$T_{r+1} = {}^{18} C_r (9x)^{18-r} \left(-\frac{1}{3\sqrt{x}} \right)^r$$

For 13th term, $r + 1 = 13$

$$r = 12$$

$$\begin{aligned} &= {}^{18}C_{12} (9x)^6 \left(-\frac{1}{3\sqrt{x}}\right)^{12} \\ &= {}^{18}C_{12} (3)^{12} \cdot x^6 \left(-\frac{1}{3}\right)^{12} \cdot (x)^{-6} \\ &= {}^{18}C_{12} (3)^{12} \cdot (-1)^{12} \cdot (3)^{-12} \\ &= {}^{18}C_{12} \\ &= 18564 \end{aligned}$$

5. Find the term independent of x in the expansion of $\left(\sqrt[3]{x} + \frac{1}{2\sqrt[3]{x}}\right)^{18}$, $x > 0$

Ans.

$$\begin{aligned} T_{r+1} &= {}^{18}C_r (\sqrt[3]{x})^{18-r} \left(\frac{1}{2\sqrt[3]{x}}\right)^r \\ &= {}^{18}C_r (x)^{\frac{18-r}{3}} \cdot \left(\frac{1}{2}\right)^r \cdot x^{-\frac{r}{3}} \\ &= {}^{18}C_r (x)^{\frac{18-r-r}{3}} \cdot \left(\frac{1}{2}\right)^r \end{aligned}$$

For independent term $\frac{18-2r}{3} = 0$

$$r = 9$$

The req. term is ${}^{18}C_9 \left(\frac{1}{2}\right)^9$



6. Find the coefficient of x^5 in the expansion of the product $(1+2x)^5(1-x)^7$

Ans.

$$(1+2x)^6(1-x)^7 = \left(1 + {}^6C_1(2x) + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 + {}^6C_5(2x)^5 + {}^6C_6(2x)^6\right)$$

$$\left(1 - {}^7C_1x + {}^7C_2(x)^2 - {}^7C_3(x)^3 + {}^7C_4(x)^4 - {}^7C_5(x)^5 + {}^7C_6(x)^6 - {}^7C_7(x)^7\right)$$

$$= (1+12x+60x^2+160x^3+240x^4+192x^5+64x^6) \cdot (1-7x+21x^2-35x^3+35x^4-21x^5+7x^6-x^7)$$

Coeff of x^5 is

$$= 1 \times (-21) + 12 \times 35 + 60 \times (-35) + 160 \times 21 + 240 \times (-7) + 192 \times 1$$

$$= 171$$

7. Compute $(98)^5$

$$\text{Ans. } (98)^5 = (100-2)^5$$

$$= {}^5C_0(100)^5 - {}^5C_1(100)^4 \cdot 2 + {}^5C_2(100)^3 \cdot 2^2$$

$$- {}^5C_3(100)^2 \cdot (2)^3 + {}^5C_4(100)(2)^4 - {}^5C_5(2)^5$$

$$= 1000000000 - 5 \times 100000000 \times 2 + 10 \times 1000000 \times 4$$

$$- 10 \times 10000 \times 8 + 5 \times 100 \times 16 - 32$$

$$= 100\ 4\ 000\ 8000 - 1000\ 8000\ 32 = 9039207968$$

8. Expand $\left(x + \frac{1}{x}\right)^6$

Ans.

$$\begin{aligned}
\left(x + \frac{1}{x}\right)^6 &= {}^6C_0(x)^6 + {}^6C_1(x^5)\left(\frac{1}{x}\right) + {}^6C_2(x^4)\left(\frac{1}{x}\right)^2 + \\
&{}^6C_3(x^3)\left(\frac{1}{x}\right)^3 + {}^6C_4(x^2)\left(\frac{1}{x}\right)^4 + {}^6C_5(x)^1\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6 \\
&= x^6 + 6x^5\left(\frac{1}{x}\right) + 15x^4\left(\frac{1}{x^2}\right) + 20x^3\left(\frac{1}{x^3}\right) + 15x^2\left(\frac{1}{x^4}\right) + 6x\left(\frac{1}{x^5}\right) + \frac{1}{x^6} \\
&= x^6 + 6x^4 + 15x^2 + 20 + \frac{6}{x^4} + \frac{1}{x^6}
\end{aligned}$$

9. Find the fourth term from the end in the expansion of $\left(\frac{x}{x^2} - \frac{x^3}{3}\right)^9$.

Ans. Fourth term from the end would be equal to $(9 - 4 + 2)^{\text{th}}$ term from the beginning

$$\begin{aligned}
T_7 = T_{6+1} &= {}^9C_6\left(\frac{3}{x^2}\right)^{9-6} \cdot \left(\frac{-x^3}{3}\right)^6 \\
&= {}^9C_6(3)^3 \cdot (x)^{-6} \cdot (x)^{18} \cdot (3)^{-6} \\
&= \frac{9!}{6!3!} \cdot (3)^{-3} \cdot (x)^{12} \\
&= \frac{28}{9} x^{12}
\end{aligned}$$

10. Find the middle term of $\left(2x - \frac{x^2}{4}\right)^9$.

Ans. $n = 9$ so there are two middle term

i.e $\left(\frac{9+1}{2}\right)^{th}$ term and $\left(\frac{9+1}{2}+1\right)^{th}$ term

$$T_5 = T_{4+1} = {}^9C_4 (2x)^{9-4} \cdot \left(\frac{-x^2}{4}\right)^4$$

$$= \frac{63}{4} x^{13}$$

$$T_6 = T_{5+1} = {}^9C_5 (2x)^{9-5} \left(\frac{-x^2}{4}\right)^5$$

$$= -{}^9C_4 (2)^4 x^4 \frac{(x)^{10}}{(4)^5}$$

$$= \frac{-63}{32} x^{14}$$

11. Find the coefficient of $a^5 b^7$ in $(a-2b)^{12}$.

Ans.

$$T_{r+1} = {}^{12}C_r (a)^{12-r} \cdot (-2b)^r$$

Put $12-r=5$

$$r=7$$

$$T_8 = {}^{12}C_7 (a)^5 \cdot (-2b)^7$$

$$= {}^{12}C_7 (a)^5 \cdot (-2)^7 b^7$$

coeff. of $a^5 b^7$ is ${}^{12}C_7 (-2)^7$



12. Find a positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6.

$$\begin{aligned} \text{Ans. } T_{r+1} &= {}^m C_r (1)^{m-r} \cdot (x)^r \\ &= {}^m C_r (x)^r \end{aligned}$$

Put $r = 2$

$$\text{ATQ } {}^m C_2 = 6$$

$$\frac{m!}{2!(m-2)!} = \frac{6}{1}$$

$$\frac{m(m-1) \cancel{(m-2)!}}{2 \times 1 \times \cancel{(m-2)!}} = \frac{6}{1}$$

$$m^2 - m = 12$$

$$m^2 - m - 12 = 0$$

$$m(m-4) = 3(m-4) = 0$$

$$(m-4)(m-3) = 0$$

$$m = 4$$

$$m = -3 \text{ (neglect)}$$

13. Show that the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1+x)^{2n-1}$.

Ans. As $2n$ is even so the expansion $(1+x)^{2n}$ has only one middle term which is

$\left(\frac{2n}{2} + 1\right)^{\text{th}}$ term i.e $(n+1)^{\text{th}}$ term

$$T_{r+1} = {}^{2n}C_r (1)^{2n-r} \cdot (x)^r$$

Coeff. of x^n is ${}^{2n}C_r$

And $(2n-1)$ is odd so two middle term

$$\left(\frac{2n-1+1}{2}\right)^{\text{th}} \text{ and } \left(\frac{2n-1+1}{2} + 1\right)^{\text{th}}$$

i.e n^{th} and $(n+1)^{\text{th}}$ term

The coefficients of these terms are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$

Now ATQ

$${}^{2n-1}C_{n-1} + {}^{2n-1}C_n = {}^{2n}C_n \left[\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r \right]$$

H.P

14. Find a if the coeff. of x^2 and x^3 in the expansion of $(3 + ax)^9$ are equal

Ans. $T_{r+1} = {}^9C_r (3)^{9-r} \cdot (ax)^r$

ATQ

$${}^9C_2 (3)^7 \cdot a^2 = {}^9C_3 (3)^6 \cdot a^3$$

$${}^9C_2 (3)^1 = {}^9C_3 \cdot a$$

$$\frac{9!}{2!7!} \times 3 = \frac{9!}{3!6!} a$$

$$\frac{3!6! \times 3}{2!7!} = a$$

$$\frac{3 \times 2 \times 1 \times 6! \times 3}{2 \times 1 \times 7 \times 6!} = a$$

$$\frac{9}{7} = a$$

15. Find $(a+b)^4 - (a-b)^4$. Hence evaluate $(\sqrt{3} + \sqrt{2})^4 + (\sqrt{3} - \sqrt{2})^4$.

$$\text{Ans. } (a+b)^4 - (a-b)^4 = \left({}^4C_0 a^4 + {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 + {}^4C_3 a b^3 + {}^4C_4 b^4 \right)$$

$$- \left(a^4 - {}^4C_1 a^3 b + {}^4C_2 a^2 b^2 - {}^4C_3 a b^3 + {}^4C_4 b^4 \right)$$

$$= 2 \left({}^4C_1 a^3 b + {}^4C_3 a b^3 \right)$$

$$= 2(4a^3 b + 4ab^3)$$

$$= 8ab(a^2 + b^2)$$

$$\text{Put } a = \sqrt{3}, \quad b = \sqrt{2}$$

$$(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 = 8\sqrt{3} \cdot \sqrt{2} (3 + 2)$$

$$= 40\sqrt{6}$$

16. Show that $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is positive integer.

$$\text{Ans. } (9)^{n+1} = (1+8)^{n+1}$$

$$\begin{aligned}
&= 1 + {}^{n+1}C_1 8^1 + {}^{n+1}C_2 \cdot 8^2 + {}^{n+1}C_3 8^3 + \dots + {}^{n+1}C_{n+1} 8^{n+1} \\
&= 1 + (n+1) \cdot 8 + 8^2 \left[{}^{n+1}C_2 + {}^{n+1}C_3 \cdot 8 + \dots + 8^{n-1} \right] \\
9^{n+1} - 8n - 9 &= 64 \left[{}^{n+1}C_2 + {}^{n+1}C_3 \cdot 8 + \dots + 8^{n-1} \right] \\
9^{n+1} - 8n - 9 &= 64k, \text{ where } k = \left[{}^{n+1}C_2 + {}^{n+1}C_3 \cdot 8 + \dots + 8^{n-1} \right]
\end{aligned}$$

17. Find the general term in the expansion of $(x^2 - yx)^{12}$

$$\begin{aligned}
\text{Ans. } T_{r+1} &= {}^{12}C_r (x^2)^{12-r} \cdot (-yx)^r \\
&= {}^{12}C_r (x)^{24-2r} \cdot (-1)^r \cdot y^r \cdot x^r \\
&= {}^{12}C_r (-1)^r \cdot y^r \cdot (x)^{24-2r}
\end{aligned}$$

18. In the expansion of $(1+a)^{m+n}$, prove that coefficients of a^m and a^n are equal.

$$\text{Ans. } T_{r+1} = {}^{m+n}C_r (1)^{m+n-r} \cdot (a)^r$$

$$T_{r+1} = {}^{m+n}C_r (a)^r \dots \dots (i)$$

Put $r = m$ and $r = n$ respectively

$$T_{m+1} = {}^{m+n}C_m a^m$$

$$\text{Coeff of } a^m \text{ is } {}^{m+n}C_m \Rightarrow \frac{(m+n)!}{m!n!}$$

Coeff of a^n is ${}^{m+n}C_m \Rightarrow \frac{(m+n)!}{n!m!}$ H.P

19. Expand $(1-x+x^2)^4$

Ans. $(1-x+x^2)^4 = [(1-x)+x^2]^4$

$$\begin{aligned}
 &= {}^4C_0(1-x)^4 + {}^4C_1(1-x)^3 \cdot (x^2) + {}^4C_2(1-x)^2 \cdot (x^2)^2 + {}^4C_3(1-x)^1(x^2)^3 + {}^4C_4(x^2)^4 \\
 &= (1-x)^4 + 4(1-x)^3 \cdot x^2 + 6(1-x)^2 \cdot x^4 + 4(1-x) \cdot x^6 + 1 \cdot x^8 \\
 &= (1-4x+6x^2-4x^3+x^4) + 4(1-3x+3x^2-x^3)x^2 + 6(1-2x+x^2)(x^4) + 4(1-x) \cdot x^6 + x^8 \\
 &= 1-4x+10x^2-16x^3+19x^4-16x^5+10x^6-4x^7+x^8
 \end{aligned}$$

20. Find the sixth term of the expansion $\left(y^{\frac{1}{2}} + x^{\frac{1}{3}}\right)^n$, if the binomial coefficient of the third term from the end is 45.

Ans. The binomial coeff of the third term from end = binomial coeff of the third term from beginning = nC_2

$${}^nC_2 = 45$$

$$\frac{n(n-1)}{1 \cdot 2} = 45$$

$$n^2 - n - 90 = 0$$

$$n = 10$$

$$T_{r+1} = {}^{10}C_r \left(y^{\frac{1}{2}}\right)^{10-r} \cdot \left(x^{\frac{1}{3}}\right)^r$$

$$r = 5$$

$$T_6 = {}^{10}C_5 \left(y^{\frac{1}{2}} \right)^5 \cdot \left(x^{\frac{1}{3}} \right)^5$$

$$= 252 y^{\frac{5}{2}} \cdot x^{\frac{5}{3}}$$

21. Find a if the 17th and 18th terms of the expansion $(2+a)^{50}$ are equal.

Ans. $T_{r+1} = {}^{50}C_r (2)^{50-r} \cdot (a)^r$

ATQ put $r = 16$ and 17

$$\Rightarrow {}^{50}C_{16} (2)^{34} \cdot a^{16} = {}^{50}C_{17} (2)^{33} \cdot a^{17}$$

$$a = \frac{{}^{50}C_{16} \times 2}{{}^{50}C_{17}}$$

$$a = 1$$

22. Find the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^6$

Ans. $T_{r+1} = {}^6C_r \left(\frac{3}{2}x^2 \right)^{6-r} \cdot \left(\frac{-1}{3x} \right)^r$

$$= {}^6C_r \left(\frac{3}{2} \right)^{6-r} \cdot (x)^{12-2r} \cdot \left(\frac{-1}{3} \right)^r \cdot x^{-r}$$

$$= {}^6C_r \left(\frac{3}{2} \right)^{6-r} \cdot \left(\frac{-1}{3} \right)^r \cdot (x)^{12-3r}$$

Put $12 - 3r = 0$

$$r = 4$$

$$= {}^6 C_4 \left(\frac{3}{2}\right)^2 \cdot \left(\frac{-1}{3}\right)^4$$

$$= \frac{5}{12}$$

23. If the coeff of $(r-5)^{\text{th}}$ and $(2r-1)^{\text{th}}$ terms in the expansion of $(1+x)^{34}$ are equal find r

$$\text{Ans. } T_{r+1} = {}^{34} C_r (1)^{34-r} \cdot (x)^r$$

$$T_{r+1} = {}^{34} C_r (x)^r \dots\dots (i)$$

Coeff are

$${}^{34} C_{r-6} \text{ and } {}^{34} C_{2r-2}$$

$$\text{ATQ } {}^{34} C_{r-6} = {}^{34} C_{2r-2}$$

$$r-6 = 2r-2$$

$$r = -4 \text{ (neglect)}$$

$$r-6 = 34 - (2r-2) \left[\begin{array}{l} \because {}^n C_r = {}^n C_p \\ r = p \text{ or } n = r + p \end{array} \right]$$

$$r = 14$$

24. Show that the coeff of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the coeff of two middle terms in the expansion of $(1+x)^{2n-1}$

Ans. As $2n$ is even so the expansion $(1+x)^{2n}$ has only one middle term which is

$$\left(\frac{2n}{2} + 1\right)^{\text{th}} \text{ i.e. } (n+1)^{\text{th}} \text{ term}$$

Coeff of x^n is ${}^{2n}C_n$

Similarly $(2n-1)$ being odd the other expansion has two middle term i.e

$$\left(\frac{2n-1+1}{2}\right)^{\text{th}} \text{ and } \left(\frac{2n-1+1}{2} + 1\right)^{\text{th}} \text{ term}$$

i.e n^{th} and $(n+1)^{\text{th}}$

The coeff are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$

$${}^{2n-1}C_{n-1} + {}^{2n-1}C_n = {}^{2n}C_n \left[\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r \right]$$

25. Find the value of r , if the coeff of $(2r+4)^{\text{th}}$ and $(r-2)^{\text{th}}$ terms in the expansion of $(1+x)^{18}$ are equal.

Ans. $T_{r+1} = {}^{18}C_r (1)^{18-r} \cdot (x)^r$

$$T_{r+1} = {}^{18}C_r x^r$$

Put $r = r-3$

And $2r+3$

ATQ ${}^{18}C_{2r+3} = {}^{18}C_{r-3}$

$$18 = 2r + 3 + r - 3$$

$$r = 6$$

26. Find the 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x \neq 0$

$$\text{Ans. } T_{r+1} = {}^{18}C_r (9x)^{18-r} \cdot \left(\frac{-1}{3\sqrt{x}}\right)^r$$

Put $r = 12$

$$T_{13} = {}^{18}C_{12} (9x)^{18-12} \cdot \left(\frac{-1}{3}\right)^{12} \cdot (x)^{\frac{-12}{2}}$$

$$= {}^{18}C_{12} (3)^{12} x^6 \cdot (3)^{-12} \cdot (-1)^{12} x^{-6}$$

$$= {}^{18}C_{12}$$



CBSE Class 12 Mathematics

Important Questions

Chapter 8

Binomial Theorem

6 Marks Questions

1. Find n , if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6} : 1$

Ans. Fifth term from the beginning in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is

$$T_{4+1} = {}^n C_4 \left(\sqrt[4]{2}\right)^{n-4} \cdot \left(\frac{1}{\sqrt[4]{3}}\right)^4$$

$$T_5 = {}^n C_4 (2)^{\frac{n-4}{4}} \cdot (3)^{-1} \dots\dots(i)$$

How fifth term from the end would be equal to $(n - 5 + 2)$ in term from the beginning

$$T_{(n-4)+1} = {}^n C_{n-4} \left(\sqrt[4]{2}\right)^{n-(n-4)} \cdot \left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$$

$$= {}^n C_{n-4} (2)^1 (3)^{\frac{n-4}{4}} \dots\dots(ii)$$

$$\text{ATQ } \frac{{}^n C_4 \cdot (2)^{\frac{n-4}{4}} (3)^{-1}}{{}^n C_{n-4} (2)^1 (3)^{\frac{n-4}{4}}} = \frac{\sqrt{6}}{1}$$

$$\frac{(2)^{\frac{n-8}{4}}}{(3)^{\frac{-(n-8)}{4}}} = (6)^{\frac{1}{2}}$$

$$(6)^{\frac{n-8}{4}} = (6)^{\frac{1}{2}}$$

$$\frac{n-8}{4} = \frac{1}{2}$$

$$\Rightarrow 2n - 16 = 4$$

$$n = 10$$

2. The coefficients of three consecutive terms in the expansion of $(1+a)^n$ are in the ratio 1:7:42. Find n

Ans. Let three consecutive terms in the expansion of $(1+a)^n$ are $(r-1)^{\text{th}}$, r^{th} and $(r+1)^{\text{th}}$ term

$$T_{r+1} = {}^n C_r (1)^{n-r} \cdot (a)^r$$

$$T_{r+1} = {}^n C_r (a)^r$$

Coefficients are

$${}^n C_{r-2}, {}^n C_{r-1} \text{ and } {}^n C_r \text{ respectively}$$

$$\text{ATQ } \frac{{}^n C_{r-2}}{{}^n C_{r-1}} = \frac{1}{7}$$

$$\Rightarrow n - 8r + 9 = 0 \dots\dots (i)$$

$$\frac{{}^n C_{r-1}}{{}^n C_r} = \frac{7}{42}$$

$$\Rightarrow n - 7r + 1 = 0 \dots (ii)$$

On solving eq. (i) and (ii) we get $n = 55$

3. The second, third and fourth terms in the binomial expansion $(x+a)^n$ are 240, 720 and 1080 respectively. Find x , a and n .

Ans. $T_2 = 240$

$${}^n C_1 x^{n-1} a = 240 \dots (i)$$

$${}^n C_2 x^{n-2} a^2 = 720 \dots (ii)$$

$${}^n C_3 x^{n-3} a^3 = 1080 \dots (iii)$$

Divide (ii) by (i) and (iii) by (ii)

We get

$$\frac{a}{x} = \frac{6}{n-1} \text{ and } \frac{a}{x} = \frac{9}{2(n-2)}$$

$$\Rightarrow n = 5$$

On solving we get

$$x = 2$$

$$a = 3$$

4. If a and b are distinct integers, prove that $a-b$ is a factor of $a^n - b^n$, whenever n is positive.

Ans. Let $a^n = (a-b+b)^n$

$$a^n = (b+a-b)^n$$

$$= {}^n C_0 b^n + {}^n C_1 b^{n-1} (a-b) + {}^n C_2 b^{n-2} (a-b)^2 + {}^n C_3 b^{n-3} (a-b)^3 + \dots + {}^n C_n (a-b)^n$$

$$a^n = b^n + (a-b) \left[{}^n C_1 b^{n-1} + {}^n C_2 b^{n-2} (a-b) + {}^n C_3 b^{n-3} (a-b)^2 + \dots + {}^n C_n (a-b)^{n-1} \right]$$

$$a^n - b^n = (a-b)k$$

Where

$${}^n C_1 b^{n-1} + {}^n C_2 b^{n-2} (a-b) + \dots + (a-b)^{n-1} = k$$

H.P

5. The sum of the coeff. Of the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^m$ being natural no. is 559. Find the term of expansion containing x^3

Ans. The coeff. Of the first three terms of $\left(x - \frac{3}{x^2}\right)^m$ are ${}^m C_0$, $(-3) {}^m C_1$ and $9 {}^m C_2$.

Therefore, by the given condition

$${}^m C_0 - 3 {}^m C_1 + 9 {}^m C_2 = 559$$

$$1 - 3m + \frac{9m(m-1)}{2} = 559$$

On solving we get $m = 12$

$$T_{r+1} = {}^{12} C_r (x)^{12-r} \left(\frac{-3}{x^2}\right)^r$$

$$= {}^{12}C_r (x)^{12-r} \cdot (-3)^r \cdot (x)^{-2r}$$

$$= {}^{12}C_r (x)^{12-3r} \cdot (-3)^r$$

$$12 - 3r = 3 \Rightarrow r = 3, \text{ req. term is } -5940 x^3$$

6. Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5 \dots (2n-1)}{n!} 2^n \cdot x^n$.

Ans. As $2n$ is even, the middle term of the expansion $(1+x)^{2n}$ is $(n+1)^{\text{th}}$ term

$$T_{n+1} = {}^{2n}C_n (1)^{2n-n} \cdot x^n$$

$$= {}^{2n}C_n x^n$$

$$= \frac{(2n)!}{n!n!} x^n$$

$$= \frac{(2n)(2n-1)(2n-2) \dots 4.3.2.1}{n!n!} x^n$$

$$= \frac{1.2.3.4 \dots (2n-2)(2n-1)(2n)}{n!n!} x^n$$

$$= \frac{[1.3.5 \dots (2n-1)][2.4.6 \dots (2n)]}{n!n!} x^n$$

$$= \frac{[1.3.5 \dots (2n-1) \cdot 2^n \cdot (1.2.3 \dots n)]}{n!n!} x^n$$

$$= \frac{[1.3.5 \dots (2n-1)] \cdot 2^n \cdot \cancel{n!}}{\cancel{n!}n!} x^n$$

$$= \frac{[1.3.5...(2n-1)] \cdot 2^n \cdot x^n}{n!}$$

7. In the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, the ratio of 7th term from the beginning to the 7th term from the end is 1:6 find n

$$\begin{aligned} \text{Ans. } T_7 &= {}^n C_6 \left(\sqrt[3]{2}\right)^{n-6} \cdot \left(\frac{1}{\sqrt[3]{3}}\right)^6 \\ &= {}^n C_6 (2)^{\frac{n-6}{3}} \cdot (3)^{-2} \dots\dots (i) \end{aligned}$$

7th term from end = $(n - 7 + 2)$ term from beginning

$$\begin{aligned} T_{n-6+1} &= {}^n C_{n-6} \left(\sqrt[3]{2}\right)^{n-n+6} \cdot \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6} \\ &= {}^n C_{n-6} (2)^2 \cdot (3)^{\frac{n-6}{-3}} \dots\dots (ii) \end{aligned}$$

ATQ

$$\frac{{}^n C_6 (2)^{\frac{n-6}{3}} \cdot (3)^{-2}}{{}^n C_{n-6} (2)^2 (3)^{\frac{6-n}{3}}} = \frac{1}{6}$$

$$\frac{(2)^{\frac{n-12}{3}}}{(3)^{\frac{12-n}{3}}} = \frac{1}{6}$$

$$(6)^{\frac{n-12}{3}} = (6)^{-1}$$

$$\frac{n-12}{3} = \frac{-1}{1}$$

$$n - 12 = -3$$

$$n = 9$$

8. If the coeff. of 5th, 6th and 7th terms in the expansion of $(1+x)^n$ are in A.P, then find the value of n .

$$\text{Ans. } T_{r+1} = {}^n C_r (1)^{n-r} \cdot (x)^r$$

$$T_{r+1} = {}^n C_r x^r \dots\dots (i)$$

Coeff of 5th, 6th, 7th terms in the expansion of $(1+x)^n$ are ${}^n C_4$, ${}^n C_5$, and ${}^n C_6$

$$\text{ATQ } 2 \cdot {}^n C_5 = {}^n C_4 + {}^n C_6$$

$$2 \cdot \frac{n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!}$$

$$n = 7, 14$$

9. If P be the sum of odd terms and Q that of even terms in the expansion of $(x+a)^n$ prove that

$$(i) \quad P^2 - Q^2 = (x^2 - a^2)^n$$

$$(ii) \quad 4PQ = (x+a)^{2n} - (x-a)^{2n}$$

$$(iii) \quad 2(P^2 + Q^2) = [(x+a)^{2n} + (x-a)^{2n}]$$

$$\text{Ans. } (x+a)^n = {}^n C_0 x^n + {}^n C_1 x^{n-1} a + {}^n C_2 x^{n-2} a^2 + \dots + {}^n C_n a^n$$

$$= t_1 + t_2 + t_3 + \dots + t_n + t_{n+1}$$

$$= (t_1 + t_3 + t_5 + \dots) + (t_2 + t_4 + t_6 + \dots)$$

$$= P + Q \dots (i)$$

$$(x - a)^n = (t_1 - t_2 + t_3 - t_4 + \dots)$$

$$= (t_1 + t_3 + t_5) - (t_2 + t_4 + t_6 \dots)$$

$$= P - Q \dots (ii)$$

$$(i) \times (ii)$$

$$P^2 - Q^2 = (x^2 - a^2)^n$$

Sq. (i) and (ii) and subt.

$$\left[(x + a)^{2n} - (x - a)^{2n} \right] = 4PQ$$

Sq. and adding we get

$$\left[(x + a)^{2n} + (x - a)^{2n} \right] = 2(P^2 + Q^2)$$

10. If three successive coeff. In the expansion of $(1 + x)^n$ are 220, 495 and 792 then find n

Ans. Let coeff are ${}^n C_{r-1}$, ${}^n C_r$, ${}^n C_{r+1}$

$$\text{ATQ } {}^n C_{r-1} = 220 \dots (i)$$

$${}^n C_r = 495 \dots (ii)$$

$${}^n C_{r+1} = 792 \dots (iii)$$

Dividing (ii) by (i)

$$\frac{{}^n C_r}{{}^n C_{r-1}} = \frac{495}{220}$$

$$\frac{n-r+1}{r} = \frac{9}{4}$$

$$4n - 13r + 4 = 0 \dots\dots (iv)$$

Dividing (iii) by (ii)

$$\frac{{}^n C_{r+1}}{{}^n C_r} = \frac{792}{495}$$

$$\frac{n-r}{r+1} = \frac{8}{5}$$

$$5n - 13r - 8 = 0 \dots\dots (v)$$

On solving (iv) and (v) we get $n = 12$